PCT

REQUEST

The undersigned requests that the present international application be processed according to the Patent Cooperation Treaty.

For receiving Office use only
International Application No.
International Filing Date
Name of receiving Office and "PCT International Application"

Applicant's or agent's file reference *(if desired) (12 characters maximum)*

Box No. I TITLE OF INVENTION

A METHOD FOR CALIBRATING THE CARRIER-PHASES OF RADIO SIGNALS FROM SATELLITES OR OTHER TRANSMITTERS BY USING A REFERENCE NETWORK

Box No. II APPLICANT ☒ This person is also inventor

Name and address: *(Family name followed by given name; for a legal entity, full official designation. The address must include postal code and name of country. The country of the address indicated in this Box is the applicant's State (that is, country) of residence if no State of residence is indicated below.)*

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Applicant's registration No. with the Office

State *(that is, country)* of nationality:

State *(that is, country)* of residence:

This person is applicant for the purposes of: ☒ all designated States ☐ all designated States except the United States of America ☐ the United States of America only ☒ the States indicated in the Supplemental Box

Box No. III FURTHER APPLICANT(S) AND/OR (FURTHER) INVENTOR(S)

Name and address: *(Family name followed by given name; for a legal entity, full official designation. The address must include postal code and name of country. The country of the address indicated in this Box is the applicant's State (that is, country) of residence if no State of residence is indicated below.)*

This person is:
☐ applicant only
☐ applicant and inventor
☐ inventor only *(If this check-box is marked, do not fill in below.)*

Applicant's registration No. with the Office

State *(that is, country)* of nationality:

State *(that is, country)* of residence:

This person is applicant for the purposes of: ☐ all designated States ☐ all designated States except the United States of America ☐ the United States of America only ☐ the States indicated in the Supplemental Box

☐ Further applicants and/or (further) inventors are indicated on a continuation sheet.

Box No. IV AGENT OR COMMON REPRESENTATIVE; OR ADDRESS FOR CORRESPONDENCE

The person identified below is hereby/has been appointed to act on behalf of the applicant(s) before the competent International Authorities as: ☐ agent ☐ common representative

Name and address: *(Family name followed by given name; for a legal entity, full official designation. The address must include postal code and name of country.)*

Telephone No.

Facsimile No.

Teleprinter No.

Agent's registration No. with the Office

☐ **Address for correspondence:** Mark this check-box where no agent or common representative is/has been appointed and the space above is used instead to indicate a special address to which correspondence should be sent.

Box No. V DESIGNATIONS

The filing of this request constitutes under Rule 4.9(a), the designation of all Contracting States bound by the PCT on the international filing date, for the grant of every kind of protection available and, where applicable, for the grant of both regional and national patents.
However,

- [ ] DE Germany is not designated for any kind of national protection
- [ ] JP Japan is not designated for any kind of national protection
- [ ] KR Republic of Korea is not designated for any kind of national protection
- [ ] RU Russian Federation is not designated for any kind of national protection

(The check-boxes above may only be used to exclude (irrevocably) the designations concerned if, at the time of filing, the international application contains in Box No. VI a priority claim to an earlier national application filed in the particular State concerned, in order to avoid the ceasing of the effect, under the national law, of this earlier national application. See the Notes to Box No. V as to the consequences of such national law provisions in these States).

Box No. VI PRIORITY CLAIM

The priority of the following earlier application(s) is hereby claimed:

Filing date of earlier application *(day/month/year)*	Number of earlier application	Where earlier application is:		
		national application: country or Member of WTO	regional application:* regional Office	international application: receiving Office
item (1) 27.02.2006	20060198	Finland		
item (2) 06.03.2006	20060219	Finland		
item (3)				

- [ ] Further priority claims are indicated in the Supplemental Box.

The receiving Office is requested to prepare and transmit to the International Bureau a certified copy of the earlier application(s) *(only if the earlier application was filed with the Office which for the purposes of this international application is the receiving Office)* identified above as:

- [ ] all items
- [x] item (1)
- [x] item (2)
- [ ] item (3)
- [ ] other, see Supplemental Box

** Where the earlier application is an ARIPO application, indicate at least one country party to the Paris Convention for the Protection of Industrial Property or one Member of the World Trade Organization for which that earlier application was filed (Rule 4.10(b)(ii)):*
....

Box No. VII INTERNATIONAL SEARCHING AUTHORITY

Choice of International Searching Authority (ISA) *(if two or more International Searching Authorities are competent to carry out the international search, indicate the Authority chosen; the two-letter code may be used):*

ISA / .EP..

Request to use results of earlier search; reference to that search *(if an earlier search has been carried out by or requested from the International Searching Authority):*

Date *(day/month/year)* Number Country *(or regional Office)*

Box No. VIII DECLARATIONS

The following declarations are contained in Boxes Nos. VIII (i) to (v) *(mark the applicable check-boxes below and indicate in the right column the number of each type of declaration):*

			Number of declarations
[ ]	Box No. VIII (i)	Declaration as to the identity of the inventor	:
[ ]	Box No. VIII (ii)	Declaration as to the applicant's entitlement, as at the international filing date, to apply for and be granted a patent	:
[ ]	Box No. VIII (iii)	Declaration as to the applicant's entitlement, as at the international filing date, to claim the priority of the earlier application	:
[x]	Box No. VIII (iv)	Declaration of inventorship (only for the purposes of the designation of the United States of America)	:
[ ]	Box No. VIII (v)	Declaration as to non-prejudicial disclosures or exceptions to lack of novelty	:

Box No. VIII (iv) DECLARATION: INVENTORSHIP (only for the purposes of the designation of the United States of America)

The declaration must conform to the following standardized wording provided for in Section 214; see Notes to Boxes Nos. VIII, VIII (i) to (v) (in general) and the specific Notes to Box No. VIII (iv). If this Box is not used, this sheet should not be included in the request.

Declaration of inventorship (Rules 4.17(iv) and 51*bis*.1(a)(iv))
for the purposes of the designation of the United States of America:

I hereby declare that I believe I am the original, first and sole (if only one inventor is listed below) or joint (if more than one inventor is listed below) inventor of the subject matter which is claimed and for which a patent is sought.

This declaration is directed to the international application of which it forms a part (if filing declaration with application).

This declaration is directed to international application No. PCT/............................ (if furnishing declaration pursuant to Rule 26*ter*).

I hereby declare that my residence, mailing address, and citizenship are as stated next to my name.

I hereby state that I have reviewed and understand the contents of the above-identified international application, including the claims of said application. I have identified in the request of said application, in compliance with PCT Rule 4.10, any claim to foreign priority, and I have identified below, under the heading "Prior Applications," by application number, country or Member of the World Trade Organization, day, month and year of filing, any application for a patent or inventor's certificate filed in a country other than the United States of America, including any PCT international application designating at least one country other than the United States of America, having a filing date before that of the application on which foreign priority is claimed.

Prior Applications: Finland 27.02.2006 20060198 and 06.03.2006 2006219

I hereby acknowledge the duty to disclose information that is known by me to be material to patentability as defined by 37 C.F.R. § 1.56, including for continuation-in-part applications, material information which became available between the filing date of the prior application and the PCT international filing date of the continuation-in-part application.

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

Name: Antti Aarne Ilmari LANGE

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Citizenship: Finnish

Inventor's Signature: [signature] Date: 27.02.2007
(The signature must be that of the inventor, not that of the agent)

Name:

Residence:
(city and either US state, if applicable, or country)

Mailing Address:

Citizenship:

Inventor's Signature: Date:
(The signature must be that of the inventor, not that of the agent)

☐ This declaration is continued on the following sheet, "Continuation of Box No. VIII (iv)".

Box No. IX CHECK LIST; LANGUAGE OF FILING

This international application contains:

(a) **on paper, the following number of sheets:**

request (including declaration sheets) :	4
description (excluding sequence listing and/or tables related thereto) :	10
claims :	2
abstract :	1
drawings :	—
Sub-total number of sheets :	17
sequence listing :	
tables related thereto :	
(for both, actual number of sheets if filed on paper, whether or not also filed in electronic form; see (c) below)	
Total number of sheets :	0

(b) ☐ **only in electronic form** (Section 801(a)(i))
- (i) ☐ sequence listing
- (ii) ☐ tables related thereto

(c) ☐ **also in electronic form** (Section 801(a)(ii))
- (i) ☐ sequence listing
- (ii) ☐ tables related thereto

Type and number of carriers (diskette, CD-ROM, CD-R or other) on which are contained the
- ☐ sequence listing: ....
- ☐ tables related thereto: ....

(additional copies to be indicated under items 9(ii) and/or 10(ii), in right column)

This international application is accompanied by the following item(s) *(mark the applicable check-boxes below and indicate in right column the number of each item):* — **Number of items**

1. ☐ fee calculation sheet :
2. ☐ original separate power of attorney :
3. ☐ original general power of attorney :
4. ☐ copy of general power of attorney; reference number, if any: .... :
5. ☐ statement explaining lack of signature :
6. ☐ priority document(s) identified in Box No. VI as item(s): .... :
7. ☐ translation of international application into *(language)*: .... :
8. ☐ separate indications concerning deposited microorganism or other biological material :
9. ☐ sequence listing in electronic form *(indicate type and number of carriers)*
   - (i) ☐ copy submitted for the purposes of international search under Rule 13*ter* only (and not as part of the international application) :
   - (ii) ☐ *(only where check-box (b)(i) or (c)(i) is marked in left column)* additional copies including, where applicable, the copy for the purposes of international search under Rule 13*ter* :
   - (iii) ☐ together with relevant statement as to the identity of the copy or copies with the sequence listing mentioned in left column :
10. ☐ tables in electronic form related to sequence listing *(indicate type and number of carriers)*
    - (i) ☐ copy submitted for the purposes of international search under Section 802(b-*quater*) only (and not as part of the international application) :
    - (ii) ☐ *(only where check-box (b)(ii) or (c)(ii) is marked in left column)* additional copies including, where applicable, the copy for the purposes of international search under Section 802(b-*quater*) :
    - (iii) ☐ together with relevant statement as to the identity of the copy or copies with the tables mentioned in left column :
11. ☐ other *(specify)*: .... :

Figure of the drawings which should accompany the abstract:

Language of filing of the international application: English

Box No. X SIGNATURE OF APPLICANT, AGENT OR COMMON REPRESENTATIVE

Next to each signature, indicate the name of the person signing and the capacity in which the person signs (if such capacity is not obvious from reading the request).

[signature]

For receiving Office use only

1. Date of actual receipt of the purported international application:
2. Drawings: ☐ received: ☐ not received:
3. Corrected date of actual receipt due to later but timely received papers or drawings completing the purported international application:
4. Date of timely receipt of the required corrections under PCT Article 11(2):
5. International Searching Authority (if two or more are competent): ISA /
6. ☐ Transmittal of search copy delayed until search fee is paid

For International Bureau use only

Date of receipt of the record copy by the International Bureau:

27.02.2007

A METHOD FOR CALIBRATING THE CARRIER-PHASES OF RADIO SIGNALS FROM SATELLITES OR OTHER TRANSMITTERS BY USING A REFERENCE NETWORK

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Abstract

Information on orbits like those of the Global Navigation Satellite Systems (GNSS) or other transmitters is collected in Near Real-Time (NRT) from global or local computing centres like those of the IGS. Carrier-phase reconstructions of the radio signals from these transmitters are received by a local reference network and forwarded operationally to a Fast Kalman Filter (FKF) processor for computing estimates of both the state and the calibration parameters accompanied with most reliable accuracy estimates. These state parameters typically include the Integrated Water Vapour (IWV) or the 3-dimensional distribution of Water Vapour (3WV) of the local troposphere and the Total Electron Content (TEC) of the local stratosphere. Precision adjustments of the carrier-phases accompanied with necessary accuracy information can then be operationally produced for the local needs of most reliable navigation, mobile positioning and warning of environmental hazards etc.

A METHOD FOR CALIBRATING THE CARRIER-PHASES OF RADIO SIGNALS FROM SATELLITES OR OTHER TRANSMITTERS BY USING A REFERENCE NETWORK

Technical Field

The invented method relates primarily to the technological convergence of Satellite Geodesy and Meteorology. The Helmert-Wolf Blocking[1] (HWB) method known from Geodesy since 1880 is expanded to **Fast Kalman Filtering (FKF)** to cover all security-critical operational applications of Kalman filtering (KF) such as Navigation, Remote Sensing and Computer Vision. Rapid fluctuations of the tropospheric water vapour and the ionospheric electron content are estimated operationally for adjusting the carrier-phases measured by a precision receiver for most reliable navigation, mobile positioning, detection of crustal movement and tsunami warning etc. Local alerts of those meteorological hazards that stem from unexpected concentrations of water vapour like tornados, thunderstorm, fog, ice formation and road slipperiness are included under the general context of Global Monitoring of Environment and Security (GMES).

Prior Art

The inventor of the Fast Kalman Filtering (FKF) reported to the scientific communities of both Satellite Geodesy and Meteorology by Lange (2001 and 2003) how his FKF formulas are closely related to the foundation-laying computations of the Helmert-Wolf Blocking (HWB) method. Single, Double and/or Triple Differences of the carrier-phases are used for sorting out Integer (lane) Ambiguities of the GNSS carrier-phase measurements in Real-Time Kinematic (RTK) and Virtual Reference Station (VRS) land surveying. A sub-decimetre level of accuracy has been achieved which is necessary for computing rough estimates of water vapour content of the atmosphere. The theory of optimal Kalman filtering (1960) is needed for building up fault-tolerance into a wide range of operational systems, including real-time imaging of atmospheric water vapour. The inventor knows no operational applications of the HWB method wherein the error covariance matrix is computed from its exact blockwise solution given by formula (3) of Lange (2001).

Summary of the Method

Large moving windows of locally linearized time-series of the carrier-phase data are analysed by the Fast Kalman Filter (FKF) processing instead of tediously sorting out the lane ambiguities in real-time. Those GNSS signal propagation effects that result from rapid variations of integrated water vapour (IWV) and total electron content (TEC) are detected or estimated depending on configuration of the satellites, reference receivers and other geophysical observations. Abrupt increases in the TEC and IWV values create detectable losses in internal consistency between all observed and simulated carrier-phases. These different effects cannot always be separated from each other neither from the clock errors of satellites nor reference receivers. Residual error variances of the carrier-phases are computed operationally using methods based on Minimum Norm Quadratic Unbiased Estimation (MINQUE) for indicating the quality and usefulness of each GNSS signal. The blockwisely computed (Lange, 2001) error covariance matrix of the estimated state and calibration parameter gives information on the accuracy of each adjustment or that the parameter contradicts Kalman's observability condition.

Best Modes of Carrying out the Method

Large moving windows of the carrier-phase data are used for maintaining Kalman's observability condition. Optimal physical and mathematical modelling is used for satisfying Kalman's controllability condition and it is monitored by computing the error variances. Adding radio frequencies and selected combinations of the GNSS signals, increasing sampling-rates and using denser receiver and meteorological observing networks improves temporal and spatial resolution. This is always made at the expense of even more rapidly increasing requirements for both computing and data transmission power.

The **Observation Equation** for a moving data-window of length L is obtained from the carrier-phase measurement $\varphi_{i,j,k,t}$ of a receiver as follows:

$$y_{i,j,k,t-l} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + g_{i,j,k,t}\, w_{k,t} + h_{i,j,t}\, c_t + e_{i,j,k,t} \qquad (1)$$

for $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $k = 1,2,\ldots,K$, $l = 0,1,2,\ldots,L-1$ and $t = L, L+1, L+2,\ldots,\infty$

where y = difference of the total carrier-phase from the distance between a satellite and a receiver
i = index of the signals (L1, L2, L3,..., G1,..., E1,..., etc.)
j = index of the satellites (GPS, Glonass and Galileo, etc.)
k = index of the receivers (or receiver sites)
l = local index of epochs for a moving data window of length L at epoch t
t = index of the epoch times (t=1, 2, 3,...)
φ = total phase of the reconstructed carrier of the i^{th} signal at epoch t
ρ = propagation distance [phase] in dry air from the j^{th} satellite to the k^{th} receiver at epoch t
τ = clock correction of the k^{th} receiver at epoch t
γ = clock correction of the j^{th} satellite at epoch t
g = slant-mapping of the IWV refractivity for the i^{th} signal from the j^{th} satellite to the k^{th} receiver at epoch t (see Slant-delay models on pages 39-49 of Kleijer (2004))
w = the IWV value for the k^{th} receiver at epoch t
h = slant-mapping of the TEC refractivity for the i^{th} signal from the j^{th} satellite

to the receiver(s) at epoch t
c = the TEC value of the receiver(s) at epoch t
e = random measurement error at epoch t; and,
m, n and K = the number of signals, satellites and receivers, respectively.

There are four **System Equations** as follows:

(2)

$$\tau_{k,t} = \tau_{k,t-1} + \zeta_{k,t}$$
$$\gamma_{j,t} = \gamma_{j,t-1} + \eta_{j,t}$$
$$\mathbf{w}_t = (A_t + dA_t)\mathbf{w}_{t-1} + \mathbf{v}_t$$
$$c_t = c_{t-1} + \xi_t$$

where

$\zeta_{k,t}, \eta_{j,t}, \xi_t$ and $\mathbf{v}_t$ = the random walk terms; respectively
$\mathbf{w}_t$ = vector $[w_{1,t}, w_{2,t}, \ldots, w_{K,t}]'$
$\mathbf{v}_t$ = vector $[v_{1,t}, v_{2,t}, \ldots, v_{K,t}]'$
A_t = state transition matrix describing the speed and direction of IWV along mean air-flow
dA_t = matrix of those state transition errors that can be adjusted by adaptive Kalman Filter.

Adaptive Fast Kalman Filtering (FKF) is applied to dense receiver and observing networks that are operated with high sampling rates (see Equations (23) and (24) on pages 12-13 in PCT/FI96/00621 of WO 97/18442).

Using the FKF processor

The **Augmented Model** of the moving sample is written out in matrix form as follows (see Equation (18) on page 11 in PCT/FI90/00122 of WO 90/13794):

(3)

$$\begin{bmatrix} y_t \\ \hat{s}_{t-1} + u_{t-1} \\ \hline y_{t-1} \\ \hat{s}_{t-2} + u_{t-2} \\ \hline \vdots \\ \hline y_{t-L+1} \\ \hat{s}_{t-L} + u_{t-L} \\ \hline \hat{C}_{t-1} + u_{C_{t-1}} \end{bmatrix} = \begin{bmatrix} H_t & & & F_t \\ I & & & \\ \hline & H_{t-1} & & F_{t-1} \\ & I & & \\ \hline & & \ddots & \vdots \\ \hline & & H_{t-L+1} & F_{t-L+1} \\ & & I & \\ \hline & & & I \end{bmatrix} \begin{bmatrix} s_t \\ s_{t-1} \\ \vdots \\ s_{t-L+1} \\ C_t \end{bmatrix} + \begin{bmatrix} e_t \\ (\hat{s}_{t-1} - s_{t-1}) - a_t \\ \hline e_{t-1} \\ (\hat{s}_{t-2} - s_{t-2}) - a_{t-1} \\ \hline \vdots \\ \hline e_{t-L+1} \\ (\hat{s}_{t-L} - s_{t-L}) - a_{t-L+1} \\ \hline (\hat{C}_{t-1} - C_{t-1}) - a_{C_t} \end{bmatrix}$$

where vectors $\mathbf{y}_t$ and $\mathbf{s}_t$ and matrix H_t represent the compositions of quantities to be **partitioned** as follows (see Equation (17) on page 10 in PCT/FI90/00122 of WO 90/13794):

(4)

$$s_t=\begin{bmatrix}b_{t,1}\\ \vdots\\ b_{t,K}\\ c_t\end{bmatrix}\quad y_t=\begin{bmatrix}y_{t,1}\\ y_{t,2}\\ \vdots\\ y_{t,K}\end{bmatrix}\quad H_t=\begin{bmatrix}X_{t,1} & & & G_{t,1}\\ & X_{t,2} & & G_{t,2}\\ & & \ddots & \vdots\\ & & X_{t,K} & G_{t,K}\end{bmatrix}$$

The following semi-analytical Fast Kalman Filtering (FKF) formulae are used for the processing (see Equation (20) on pages 11-12 in PCT/FI90/00122 of WO 90/13794):

(5)

$$\hat{s}_{t-l}=\left\{X'_{t-l}V^{-1}_{t-l}X_{t-l}\right\}^{-1}X'_{t-l}V^{-1}(y_{t-l}-G_{t-l}\hat{c}_t)\quad \text{for } l=0,1,2,\dots,L-1$$

$$\hat{c}_t=\left\{\sum_{l=0}^{L}G'_{t-l}R_{t-l}G_{t-l}\right\}^{-1}\sum_{l=0}^{L}G'_{t-l}R_{t-l}y_{t-l}$$

where, for $l=0,1,2,\dots,L-1$,

$$R_{t-l}=V^{-1}_{t-l}\left\{I-X_{t-l}\left\{X'_{t-l}V^{-1}_{t-l}X_{t-l}\right\}^{-1}X'_{t-l}V^{-1}_{t-l}\right\}$$

$$V_{t-l}=\begin{bmatrix}Cov(e_{t-l}) & \\ & Cov\left\{(\hat{s}_{t-l-1}-s_{t-l-1})-a_{t-l}\right\}\end{bmatrix}$$

$$y_{t-l}=\begin{bmatrix}y_{t-l}\\ \hat{s}_{t-l-1}+u_{t-l-1}\end{bmatrix}$$

$$X_{t-l}=\left[\frac{H_{t-l}}{I}\right]$$

$$G_{t-l}=\left[\frac{F_{t-l}}{\ }\right]$$

and, i.e. for $l=L$,

$$R_{t-L}=V^{-1}_{t-L}$$

$$V_{t-L}=Cov\left\{(\hat{c}_{t-1}-c_{t-1})-a_{c_t}\right\}$$

$$y_{t-L}=\hat{c}_{t-1}+u_{c_{t-1}}$$

$$G_{t-L}=I.$$

The Hybrid Windfinding Algorithm (HWA) reported in Paper 5 of Lange (1999) computes recursively the Best Linear Unbiased Estimates (BLUE) in real-time for the clock corrections of the receivers (k=1, 2,…, K) and the satellites (j=1, 2,.., n), the values (k=1, 2,…, K) of integrated water vapour (IWV) and the total value of ionospheric electron content (TEC). Their accuracies depend on both information and the degree of over-determination that the **Augmented Model** (3) has at each epoch time t. The accuracies of all the calibration parameters and/or the adjusted carrier-phases:

$$y_{i,j,k,t} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + g_{i,j,k,t}\, w_{k,t} + h_{i,j,t}\, c_t \qquad (6)$$

for $i = 1,2,\ldots,m$, $j=1,2,\ldots,n$, $k=1,2,\ldots,K$, and $t=L, L+1, L+2,\ldots,\infty$

are estimated in real-time or in NRT by using an outcome from C. R. Rao's MINQUE theory (see e.g. Equation (23) on page 19 in Paper 5 of Lange (1999)).

Firstly, in order to specify vectors $\mathbf{y}_t$ and $\mathbf{s}_t$ and matrix $\mathbf{H}_t$ the following logical insertions are made in Equations (4):

$\mathbf{c}_t$:= [empty]
and for all k=1,2,...,K:

$\mathbf{b}_{t,k} := \tau_{k,t}$
$\mathbf{y}_{t,k} := [y_{1,1,k,t}, y_{2,1,k,t},\ldots, y_{m,1,k,t},\ y_{1,2,k,t}, y_{2,2,k,t},\ldots, y_{m,2,k,t}, \ldots,\ y_{1,n,k,t}, y_{2,n,k,t},\ldots, y_{m,n,k,t}, \tau_{k,t-1}]'$
$X_{t,k} := [1, 1, 1,\ldots,1, 1]'$ and $G_{t,k}$:= [empty];

so that $\mathbf{y}_t = [\mathbf{y}'_{t,1}, \mathbf{y}'_{t,2},\ldots, \mathbf{y}'_{t,K}]'$, $\mathbf{s}_t = [\tau_{1,t}, \tau_{2,t},\ldots, \tau_{K,t}]$ and $\mathbf{H}_t = \mathrm{diag}(X_{t,1}, X_{t,2},\ldots, X_{t,K})$.

Thereafter, the following logical insertions are made in the **Augmented Model** of Equation (3):

$\mathbf{F}_t := [\mathrm{diag}(\mathbf{f}_{t,1}, \mathbf{f}_{t,2},\ldots, \mathbf{f}_{t,n}), \mathrm{diag}(\mathbf{g}_{t,1}, \mathbf{g}_{t,2},\ldots, \mathbf{g}_{t,K}), [\mathbf{h}'_{t,1}, \mathbf{h}'_{t,2},\ldots, \mathbf{h}'_{t,K}]']$

where
$\mathbf{f}_{t,j} = [1, 1, 1,\ldots, 1, 0]'$
$\mathbf{g}_{t,v} = [\mathbf{g}'_{t,v,1}, \mathbf{g}'_{t,v,2},\ldots, \mathbf{g}'_{t,v,K}]'$
$\mathbf{g}_{t,k} = [g_{1,1,k,t}, g_{2,1,k,t},\ldots, g_{m,1,k,t},\ g_{1,2,k,t}, g_{2,2,k,t},\ldots, g_{m,2,k,t}, \ldots,\ g_{1,n,k,t}, g_{2,n,k,t},\ldots, g_{m,n,k,t}, 0]'$
where $g_{i,j,k,t}$ = slant-path refractivity of IWV for the i[th] signal from the j[th] satellite to the k[th] receiver
and
$\mathbf{h}_{t,k} = [h_{1,1,t}, h_{2,1,t}, \ldots, h_{m,1,t},\ h_{1,2,t}, h_{2,2,t}, \ldots, h_{m,2,t}, \ldots,\ h_{1,n,t}, h_{2,n,t}, \ldots, h_{m,n,t}, 0]'$
where $h_{i,j,t}$ = slant-path refractivity of TEC for the i[th] signal from the j[th] satellite to the k[th] receiver;

and,

$\hat{\mathbf{s}}_t = \mathbf{u}_t$:= [empty], $\mathbf{u}_c$:=0 and $\mathbf{C}_t := [\gamma_{1,t}, \gamma_{2,t},\ldots, \gamma_{n,t},\ w_{1,t}, w_{2,t},\ldots, w_{K,t}, c_t]'$;
so that for Equations (5): $\mathbf{y}_{t-l} = y_{t-l}$, $\mathbf{X}_{t-l} = [\mathbf{H}_{t-l}]$ and $\mathbf{G}_{t-l} = [\mathbf{F}_{t-l}]$
where vector $\hat{\mathbf{C}}_t$ with the hat (^) on top of it gives the BLUE estimates for tomography etc.

This method above can be extended to the 3- (or 4-) dimensional tomography where also vertical (and temporal) variations of all atmospheric constituents are explicitly taken into account. This is made at the expense of extra lapsed time required for collecting and properly processing much more data (see Equations (26-29) on pages 12-13 in PCT/FI93/00192 of WO 93/22625):

The **Observation Equation** for a moving data-window of length L is then obtained from the carrier-phase measurement $\varphi_{i,j,k,t}$ of a receiver as follows:

$$y_{i,j,k,t-l} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + \mathbf{g}'_{j,k,t}\mathbf{w}_t + h_{i,j,t}c_t + e_{i,j,k,t} \qquad (1t)$$

for $i=1,2,\ldots,m$, $j=1,2,\ldots,n$, $k=1,2,\ldots,K$, $l=0,1,2,\ldots,L-1$ and $t=L, L+1, L+2,\ldots,\infty$

where y = difference of the total carrier-phase from the distance between a satellite and a receiver
- i = index of the signals (L1, L2, L3,... , G1,... , E1,... , etc.)
- j = index of the satellites (GPS, Glonass and Galileo, etc.)
- k = index of the receivers (or receiver sites)
- l = local index of epochs for a moving data window of length L at epoch t
- t = index of the epoch times (t=1, 2, 3,...)
- φ = total phase of the reconstructed carrier of the ith signal at epoch t
- ρ = propagation distance [phase] in dry air from the jth satellite to the kth receiver at epoch t
- τ = clock correction of the kth receiver at epoch t
- γ = clock correction of the jth satellite at epoch t
- $\mathbf{g}$ = vector of the slant-path 3WV refractivity values of pixel volumes from the jth satellite to the kth receiver at epoch t (see Slant-delay models on pages 39-49 of Kleijer (2004))
- $\mathbf{w}$ = vector of the 3WV values of pixel volumes at epoch t
- h = slant-mapping of the TEC refractivity for the ith signal from the jth satellite to the receiver network(s) at epoch t
- c = the TEC value of the receiver network(s) at epoch t
- e = random measurement error at epoch t; and,
- m, n, K and V = the number of signals, satellites, receivers and pixel volumes, respectively.

There are four **System Equations** as follows: (2t)

$$\tau_{k,t} = \tau_{k,t-1} + \zeta_{k,t}$$
$$\gamma_{j,t} = \gamma_{j,t-1} + \eta_{j,t}$$
$$\mathbf{w}_t = (A_t + dA_t)\mathbf{w}_{t-1} + \mathbf{v}_t$$
$$c_t = c_{t-1} + \xi_t$$

where
- $\zeta_{k,t}$, $\eta_{j,t}$, $\mathbf{v}_t$ and ξ_t = the random walk terms; respectively
- $\mathbf{w}_t$ = vector $[w_{1,t}, w_{2,t},\ldots, w_{V,t}]'$
- $\mathbf{v}_t$ = vector $[v_{1,t}, v_{2,t},\ldots, v_{V,t}]'$
- A_t = state transition matrix describing advection of the 3WV values in the air-mass
- dA_t = matrix of the state transition errors to be adjusted by adaptive Kalman Filtering.

Matrix A is a tangent-linear type of an approximation of the Numerical Weather Prediction (NWP) model that is applied in the data assimilation of the 3WV values at epoch t for obtaining them from their previous values at epoch t-1 (see Equations (26-29) on pages 12-13 in PCT/FI93/00192 of WO 93/22625). Matrix dA is approximated by a vector **r** that is estimated by adaptive Fast Kalman Filtering (FKF) (see Equations (22-24) on pages 12-13 in PCT/FI96/00621 of WO 97/18442).

Using the FKF processor

The **Augmented Model** of the moving sample is written out in matrix form as follows (see Equation (24) on page 13 in PCT/FI96/00621 of WO 97/18442):

(3t)

$$\begin{bmatrix} y_t \\ A\hat{s}_{t-1}+Bu_{t-1} \\ \hline y_{t-1} \\ A\hat{s}_{t-2}+Bu_{t-2} \\ \hline \vdots \\ \hline y_{t-L+1} \\ A\hat{s}_{t-L}+Bu_{t-L} \\ \hline A\hat{c}_{t-1}+Bu_{c\,t-1} \end{bmatrix} = \begin{bmatrix} \begin{matrix} H_t \\ I \end{matrix} & & & & \begin{matrix} F^y_t \\ F^s_t \end{matrix} & M_{t-1} \\ & \begin{matrix} H_{t-1} \\ I \end{matrix} & & & \begin{matrix} F^y_{t-1} \\ F^s_{t-1} \end{matrix} & M_{t-2} \\ & & \ddots & & \vdots & \vdots \\ & & & \begin{matrix} H_{t-L+1} \\ I \end{matrix} & \begin{matrix} F^y_{t-L+1} \\ F^s_{t-L+1} \end{matrix} & M_{t-L} \\ & & & & & I \end{bmatrix} \begin{bmatrix} s_t \\ s_{t-1} \\ \vdots \\ s_{t-L+1} \\ \hline c_t \end{bmatrix} + \begin{bmatrix} e_t \\ A(\hat{s}_{t-1}-s_{t-1})-a_t \\ \hline e_{t-1} \\ A(\hat{s}_{t-2}-s_{t-2})-a_{t-1} \\ \hline \vdots \\ \hline e_{t-L+1} \\ A(\hat{s}_{t-L}-s_{t-L})-a_{t-L+1} \\ \hline A(\hat{c}_{t-1}-c_{t-1})-a_{c_t} \end{bmatrix}$$

where vectors y_t and s_t and matrix H_t represent the compositions of quantities to be **partitioned** as follows (see Equation (17) on page 8 in PCT/FI96/00621 of WO 97/18442):

(4t)

$$s_t = \begin{bmatrix} b_{t,1} \\ \vdots \\ b_{t,K} \\ c_t \end{bmatrix} \quad y_t = \begin{bmatrix} y_{t,1} \\ y_{t,2} \\ \vdots \\ y_{t,K} \end{bmatrix} \quad H_t = \begin{bmatrix} X_{t,1} & & & & G_{t,1} \\ & X_{t,2} & & & G_{t,2} \\ & & \ddots & & \vdots \\ & & & X_{t,K} & G_{t,K} \end{bmatrix}$$

The following semi-analytical Fast Kalman Filtering (FKF) formulae are used for the processing (see Equations (25) on page 14 in PCT/FI96/00621 of WO 97/18442):

(5t)

$$\hat{s}_{t-l} = \left\{X'_{t-l}V^{-1}_{t-l}X_{t-l}\right\}^{-1}X'_{t-l}V^{-1}(y_{t-l}-G_{t-l}\hat{c}_t) \quad \textbf{for } l=0,1,2,\ldots,L-1$$

$$\hat{c}_t = \left\{\sum_{l=0}^{L} G'_{t-l}R_{t-l}G_{t-l}\right\}^{-1} \sum_{l=0}^{L} G'_{t-l}R_{t-l}y_{t-l}$$

where, for $l=0,1,2,\ldots,L-1$,

$$R_{t-l} = V^{-1}_{t-l}\left\{I - X_{t-l}\left\{X'_{t-l}V^{-1}_{t-l}X_{t-l}\right\}^{-1}X'_{t-l}V^{-1}_{t-l}\right\}$$

$$V_{t-l} = \begin{bmatrix} Cov(e_{t-l}) & \\ & Cov\left\{A_{t-l}(\hat{s}_{t-l-1} - s_{t-l-1}) - a_{t-l}\right\} \end{bmatrix}$$

$$y_{t-l} = \begin{bmatrix} y_{t-l} \\ A_{t-l}\,\hat{s}_{t-l-1} + B_{t-l}\,u_{t-l-1} \end{bmatrix}$$

$$X_{t-l} = \left[\frac{H_{t-l}}{I}\right]$$

$$G_{t-l} = \left[\frac{F^{y}_{t-l}}{F^{s}_{t-l}}\ \overline{M_{t-l-1}}\right]$$

and, i.e. for $l = L$,

$$R_{t-L} = V^{-1}_{t-L}$$

$$V_{t-L} = Cov\left\{A_c(\hat{C}_{t-1} - C_{t-1}) - a_{c_t}\right\}$$

$$y_{t-L} = A_c\hat{C}_{t-1} + B_c u_{c_{t-1}}$$

$$G_{t-L} = I.$$

The HWA algorithm in Paper 5 of Lange (1999) computes recursively the Best Linear Unbiased Estimates (BLUE) in real-time for the clock corrections of the receivers (k=1, 2,..., K) and the satellites (j=1, 2,.., n), the voxel (pixel volume) values (v=1, 2,..., V) of water vapour (3WV) and the value of ionospheric electron content (TEC). Their accuracies depend on both the information and the degree of over-determination that the **Augmented Model** (3t) contains at each epoch time t. The accuracies of all the calibration parameters and/or the adjusted carrier-phases:

$$y_{i,j,k,t-l} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + \mathbf{g}'_{j,k,t}\mathbf{w}_t + h_{i,j,t}\,c_t \qquad (6t)$$

for i =1,2,...,m, j=1,2,...,n, k=1,2,...,K, and t=L, L+1,L+2,...,∞

are estimated in real-time or in NRT by using an outcome from C. R. Rao's MINQUE theory (see e.g. Equation (23) on page 19 in Paper 5 of Lange (1999)).

Firstly, in order to specify vectors $\mathbf{y}_t$ and $\mathbf{s}_t$ and matrix $\mathbf{H}_t$ the following logical insertions are made in Equations (4t):
$\mathbf{c}_t$:= [empty]
and for all k=1,2,...,K:
$\mathbf{b}_{t,k} := \tau_{k,t}$

$\mathbf{y}_{t,k} := [y_{1,1,k,t}, y_{2,1,k,t},..., y_{m,1,k,t}, \quad y_{1,2,k,t}, y_{2,2,k,t},..., y_{m,2,k,t}, \ ..., \quad y_{1,n,k,t}, y_{2,n,k,t},..., y_{m,n,k,t}, \tau_{k,t-1}]'$
$X_{t,k} := [1, 1, 1,...,1, 1]'$ and $G_{t,k} :=$ [empty] ;
so that $\mathbf{y}_t = [\mathbf{y}'_{t,1}, \mathbf{y}'_{t,2},..., \mathbf{y}'_{t,K}]'$, $\mathbf{s}_t = [\tau_{1,t}, \tau_{2,t},..., \tau_{K,t}]$ and $\mathbf{H}_t = \text{diag}(X_{t,1}, X_{t,2},..., X_{t,K})$.

Thereafter, the following logical insertions are made in the **Augmented Model** of Equation (3t):

$\mathbf{F}_t := [\text{diag}(\mathbf{f}_{t,1}, \mathbf{f}_{t,2},..., \mathbf{f}_{t,n}), [\mathbf{g}_{t,1}, \mathbf{g}_{t,2},..., \mathbf{g}_{t,V}], [\mathbf{h}'_{t,1}, \mathbf{h}'_{t,2},..., \mathbf{h}'_{t,K}]']$

where
$\mathbf{f}_{t,j} = [1, 1, 1,..., 1, 0]'$
$\mathbf{g}_{t,v} = [\mathbf{g}'_{t,v,1}, \mathbf{g}'_{t,v,2},..., \mathbf{g}'_{t,v,K}]'$
$\mathbf{g}_{t,v,k} = [g_{1,1,k,v,t}, g_{2,1,k,v,t},..., g_{m,1,k,v,t}, \ g_{1,2,k,v,t}, g_{2,2,k,v,t},..., g_{m,2,k,v,t}, ..., g_{1,n,k,v,t}, g_{2,n,k,v,t},..., g_{m,n,k,v,t}, 0]'$
where $g_{i,j,k,v,t}$ = slant-path refractivity of 3WV for voxel v if the ith signal from the jth satellite goes to the kth receiver through it at epoch time t else =0
and
$\mathbf{h}_{t,k} = [h_{1,1,t}, h_{2,1,t}, ..., h_{m,1,t}, \quad h_{1,2,t}, h_{2,2,t}, ..., h_{m,2,t}, \ ..., \ h_{1,n,t}, h_{2,n,t}, ..., h_{m,n,t}, \ 0]'$
where $h_{i,j,t}$ = slant-path refractivity of TEC for the ith signal from the jth satellite to the kth receiver t;

and,

$\hat{\mathbf{s}}_t = \mathbf{Bu}_t :=$ [empty], $\mathbf{u}_c := 0$ and $\mathbf{C}_t := [\gamma_{1,t}, \gamma_{2,t},..., \gamma_{n,t}, w_{1,t}, w_{2,t},..., w_{V,t}, c_t, \mathbf{r}'_t]'$ and $\mathbf{r}_t$ = a vector of selected elements of matrix dA_t of Equation (23) as specified on pages 12-13 of PCT/FI96/00192; so that for Equations (5t): $\mathbf{y}_{t-l} = \mathbf{y}_{t-l}$, $\mathbf{X}_{t-l} = [\mathbf{H}_{t-l}]$, $\mathbf{G}_{t-l} = [\mathbf{F}_{t-l}, \mathbf{0}]$ and $\mathbf{G}_{t-L} = \text{diag}(\mathbf{I}, \mathbf{M}_{t-L})$

where vector $\hat{\mathbf{C}}_t$ with the hat (^) on top of it gives the BLUE estimates for tomography etc.

This method can be extended to 3- or 4-dimensional data-assimilation where the temporal variation of atmospheric constituents is taken more explicitly into account at the expense of extra lapsed time that is required for collecting and processing more data by other computing methods (see Equations (26-29) on pages 12-13 in PCT/FI93/00192 of WO 93/22625). There are similar applications of this invention which should not be excluded if the claims do not specifically say so.

References

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CLAIMS

Claim 1. A method for adjusting model and/or calibration parameters of a sensor system that is equipped with said model of external events where sensor output units of said system provide signals in response to said external events and said method makes use of Fast Kalman Filtering (FKF) that comprises the following steps:

a) provides a data base unit for storing information on:

- a plurality of test point sensor output signal values for some of said sensors and a plurality of values for external events that correspond to said test point sensor output signal values and/or simultaneous time series of sensor output signal values from adjacent sensors for comparison;

- values of said sensor output signals, values of said model and/or calibration parameters and values of said external events that correspond to a situation; and,

- controls of said sensors and/or forcings of said external events that correspond to said situation;

b) provides a logic unit for accessing both said sensor signal output values and said model and/or calibration parameter values, where said logic unit has both a two-way communications link to said data base unit and the capability of computing initial values for unknown model and/or calibration parameters;

c) provides said sensor output signal values from said sensors, as available, to said logic unit;

d) provides information on said controls of said sensors and/or said forcings of said external events to said data base unit;

e) accesses current values of both model and/or calibration parameters and state transition matrices, and computes updated values of said model and/or calibration parameters by using semi-analytical FKF-formulas that are obtained by applying Frobenius Inversion Formula (26):

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1}+A^{-1}BH^{-1}CA^{-1} & -A^{-1}BH^{-1} \\ -H^{-1}CA^{-1} & H^{-1} \end{bmatrix}$$

where $H = D - CA^{-1}B$

for solving the resulting Normal Equation system by exploiting a block-diagonalization of the covariance matrix of the residual errors $[\mathbf{e}_t', (A(\hat{\mathbf{s}}_{t-1} - \mathbf{s}_{t-1}) - \mathbf{a}_t)']'$ of Augmented Model (8):

$$\begin{bmatrix} \mathbf{y}_t \\ A\hat{\mathbf{s}}_{t-1} + B\mathbf{u}_{t-1} \end{bmatrix} = \begin{bmatrix} H_t \\ I \end{bmatrix} \mathbf{s}_t + \begin{bmatrix} \mathbf{e}_t \\ A(\hat{\mathbf{s}}_{t-1} - \mathbf{s}_{t-1}) - \mathbf{a}_t \end{bmatrix}$$

by applying factors F, F^y, F^s or M to said Augmented Model, in said logic unit, for said situation; and where the improvement comprises computing of a local FKF solution of Augmented Model (3 or 3t) for carrier-phases of signals from satellites or other transmitters;

f) controls stability of said FKF-filtering by monitoring accuracy estimates of said updated values of model and/or calibration parameters, in said logic unit, and indicates needs for sensor output signal values, test point data, sensor comparisons or a system reconfiguration;

g) adjusts said model and/or calibration parameter values if stable updates are available.