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Paper title: Using Helmert-Wolf blocking for diagnosis & treatment of GNSS errors

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Abstract: Safe traffic needs new standards to secure ultra-reliable and precise navigation. The sufficient amount and density of information for cruising worldwide with an autonomous vehichle is obtained by using numerous GNSS and LBS signals. An uncompromised level of integrity is needed. The potential hazards of Selective Availability (SA), interference or spoofing are taken care by analysing the internal consistency of these GNSS, pseudolite, beacon, radar, lidar, sonar, gyro, odometer, INS unit, etc. signals for finding their outliers and remedies.. The Minimum Norm Quadratic Unbiased Estimation (MINQUE) gives the signal error variances. Optimal transport speeds are achieved using all available signals. Conventional Kalman recursions were replaced by Helmert-Wolf blocking (HWb) from Geodesy. The navigation accuracies are computed in realtime.

Draft paper:

INTRODUCTION

Foreign GNSS signals will soon need a Federal Communications Committee (FCC) authorization for use in the United States. Multi-GNSS receivers needs to be certified in order to be legally used in safety-critical transports. The Receiver Autonomous Integrity Monitoring (RAIM) must be applied. Sophisticated A Posteriori Multipath Estimator (APME) methods needs to be developed.

The Fault Detection and Exclusion (FDE) techniques can improve robustness in the presence of a signal failure. The difference between an observed and the expected signal is divided by its standard deviation. This ratio is compared with a threshold value for a small probability of false alarms. However, the Best Linear Unbiased Estimation (BLUE) of the expected signal values requires extremely fast computing from the Multi-GNSS receiver. The Real-Time Kinematic (RTK) and Virtual Reference Station (VRS) land surveying makes use of the sparse-matrix method of the Helmert-Wolf blocking (HWb). It is fast enough and applies also to the most stringent optimal Kalman filtering that is required of any truly safe receiver.

FAST KALMAN FILTERING (FKF)

Helmert-Wolf blocking (HWb)

The linearized regression equation system (1) of the signal data from all receivers and instruments is written out in the Canonical Block-Angular (CBA) form as outlined by F. R. Helmert in 1880 (see Image 1) as follows:

$$\mathbf{y} = \mathbf{H}\,\mathbf{s} + \mathbf{e} \tag{1}$$

Matrix **H** is typically an extremely huge Jacobian matrix where its submatrices are moderately sized Jacobians of the state and the calibration parameters. Wolf's formulas (2) for computing all these state and calibration parameters are (see Image 2):

$$\mathbf{b}_k = (\mathbf{X}_k'\mathbf{X}_k)^{-1}\mathbf{X}_k'(\mathbf{y}_k - \mathbf{G}_k\mathbf{c}) \quad \text{for} \quad k = 1, 2, \ldots, K; \tag{2}$$
$$\mathbf{c} = (\textstyle\sum \mathbf{G}_k'\mathbf{R}_k\mathbf{G}_k)^{-1}\sum \mathbf{G}_k'\mathbf{R}_k\,\mathbf{y}_k$$

The error covariance matrix of all the estimated parameters $\mathbf{s} = [\,\mathbf{b}_1', \mathbf{b}_2', \ldots, \mathbf{b}_K', \mathbf{c}'\,]'$ will thereafter take the following simple and also numerically exploitable form (see Image 3):

$$\mathrm{Cov}\,(\mathbf{s} - \mathrm{E}\,(\mathbf{s})) = \mathrm{E}\,[\mathbf{s} - \mathrm{E}\,(\mathbf{s})]\,[\mathbf{s} - \mathrm{E}\,(\mathbf{s})]' \tag{3}$$

The required normalization of the error vectors **e** of the *n* signals and of the *m* calibration parameters has to be made by suitably transforming the measurement equations (1). Multivariate statistical methods such as a generalized Canonical Correlation Analysis (gCCA) may also be needed for the elimination of an unwanted heteroscedasticity from the error variances. However, it has been an overly difficult task to estimate the error covariance matrix $\mathrm{Cov}\,(\mathbf{e}) = \mathrm{E}\,(\mathbf{e}\ \mathbf{e}')$ for operational purposes.

Estimating the true Precision

C. R. Rao's Minimum Norm Quadratic Unbiased Estimation (MINQUE) provides theoretically the most efficient estimation of the error covariance matrix $\mathrm{Cov}\,(\mathbf{e}) = \mathrm{E}\,(\mathbf{e}\ \mathbf{e}')$ of the signals and their calibration data. However, the reliable mathematical solution calls for very large matrix inversions. Existing Floating-Point Units (FPU) cannot cope with it due to their too short mantissas. Fortunately, the patented FKF method uses the semi-analytic sparse-matrix techniques of HWb and solves this problems for all operational applications of precise navigation and probe tracking.

Vector $\boldsymbol{\sigma}$ of the *n+m* error variances of all the signal and calibration input data can now be estimated according to Rao's MINQUE theory from the simple-looking formula (see Image 4):

$$\boldsymbol{\sigma} = [\,\sigma_1^2,\ \sigma_2^2, \ldots, \sigma_N^2\,]' = \mathbf{F}^{-1}\,\mathbf{q} \tag{4}$$

where

$\boldsymbol{\sigma}$ = vector of N unknown error variances

N = *n+m* = total number of the n different signals and the m calibration inputs

$\mathbf{q}$ = vector of the N sums of squared computed residuals $\mathbf{e} = [\,\mathbf{e}_1', \mathbf{e}_2', \ldots, \mathbf{e}_K', \mathbf{e}_{K+1}'\,]'$

$\mathbf{F}$ = NxN square matrix.

The residuals computed from the vector $\mathbf{y} = [\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_K', \mathbf{y}_{K+1}']'$ are called the *innovations sequence* of Kalman filtering. The hardest task here is to compute matrix $\mathbf{F}$ in real-time. The quadratic estimators of these error variances may, in theory, obtain negative values as strictly unbiased. Slightly easier to compute estimates are obtained using an Almost Unbiased Estimation (AUE) approximation for the matrix $\mathbf{F}$. Such AUE variances are non-negative. These MINQUEs can be computed operationally by applying the FKF method as follows:

$$\mathbf{q} = [\,\mathbf{y'RT}_1\mathbf{y},\ \mathbf{y'RT}_2\mathbf{y},\ \mathbf{y'RT}_3\mathbf{y}, \ldots, \mathbf{y'RT}_N\mathbf{y}\,]' \tag{5}$$

$\mathbf{F}$ = matrix $\{\,\mathrm{trace}\,(\mathbf{RT}_i\,\mathbf{RT}_j)\}$ where i = 1, 2, 3,... , N and j = 1, 2, 3,... , N

$\mathbf{R} = \mathbf{I} - \mathbf{H}\ \mathrm{Cov}\,(\mathbf{s} - \mathrm{E}\,(\mathbf{s}))\,\mathbf{H}'$

$\mathbf{T}_i$ = gigantic sparse-matrix (i = 1, 2, 3,... , N).

The sparse matrix $\mathbf{T}_i$ is diagonal for a typical navigation application. Its scalar elements are Kronecker's deltas δ that indicate which data from one of the N different signals are selected e.g.

$\mathbf{T}_i = \mathrm{diag}\,\{\,\mathrm{diag}\,(\boldsymbol{\delta}_{i,1}, \boldsymbol{\delta}_{i,2}, \ldots, \boldsymbol{\delta}_{i,h\,1}),\ \mathrm{diag}\,(\boldsymbol{\delta}_{i,1}, \boldsymbol{\delta}_{i,2}, \ldots, \boldsymbol{\delta}_{i,h\,2}), \ldots, \mathrm{diag}\,(\boldsymbol{\delta}_{i,1}, \boldsymbol{\delta}_{i,2}, \ldots, \boldsymbol{\delta}_{i,h\,K+1})\,\}$

where $\boldsymbol{\delta}_{i,h} = 1$ if the place h in subdiagonal k (k=1,2,..., K+1) is entitled to the input data from the signal i; otherwise $\boldsymbol{\delta}_{i,h} = 0$. Here we will have at least $n * K + m$ such diagonal elements.

The CBA equations (1) should be normalized again and again using the improving new error variance estimates as follows:

$$\mathrm{Cov}\,(\mathbf{e}) = \sum \boldsymbol{\sigma}_i^2\,\mathbf{T}_i \tag{6}$$

where the summation index i runs from 1 to N. This kind of accuracy estimation does not, in the long run, depend on the given a priori values as long as the FKF process is kept optimal in all other respects. Equations (3) - (6) will then provide objective accuracy estimates as they are derived from the truly measured integrity of the entire system.

CONCLUDING REMARKS

The theory of optimal Kalman filtering provides the proper means of updating repeatedly the receiver positions, instrumental calibration drifts, system model and environment parameters. The FKF processing exploits large moving data windows. These must be kept long to satisfy Kalman's observability and controlabilty conditions also for those calibration parameters that are involved in safety-critical navigation. The straightforward way of controlling them is to monitor their true realtime accuracy by MINQUE. However, no conventional Kalman filter is able to deal with the excessive computational burden because its numerical complexity is proportional much more than to the cube of the number of the input signals and variables. Fortunately, the numerical complexity of FKF is roughly proportional only to the square. Sophisticated RAIM, FDE and APME techniques are developed for full exploitation of the numerous new GNSS, position location and IMU signals. The forthcoming ultra-reliable hybrid precision receivers will improve cruise control and piloting of robots and vehicles. Our simulations demonstrate how combining many different signals improves precision and robustness for more intelligent traffic and transports.

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Image 1:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \\ \underline{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \\ & & & & I \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \\ \mathbf{e}_C \end{bmatrix} \qquad (1)$$

i.e.

$$\mathbf{y} = \mathbf{H} \quad \mathbf{s} + \mathbf{e}$$

Image 2:

$$\mathbf{b}_k = (X_k' X_k)^{-1} X_k' (\mathbf{y}_k - G_k \mathbf{c}) \quad \text{for } k = 1, 2, \ldots, K;$$

$$\mathbf{c} = \left(\sum^{K+1} G_k' R_k G_k\right)^{-1} \sum^{K+1} G_k' R_k \mathbf{y}_k \qquad (2)$$

where

$R_k = I - X_k (X_k' X_k)^{-1} X_k'$ = residual operator for data block k;

$R_{K+1} = I$ = "residual operator" for the a priori calibration data $\mathbf{y}_{K+1}$;

$G_{K+1} = I$ = "Jacobian matrix" for the a priori calibration data $\mathbf{y}_{K+1}$;

Image 3:

$$\mathrm{Cov}\,(s - E(s)) = E\,[\,s - E(s)]\,[\,s - E(s)]' = \qquad (3)$$

$$= \begin{bmatrix} C_1 + D_1SD_1' & D_1SD_2' & \cdots & D_1SD_K' & -D_1S \\ D_2SD_1' & C_2 + D_2SD_2' & & D_2SD_K' & -D_2S \\ \vdots & & \ddots & \vdots & \vdots \\ D_KSD_1' & D_KSD_2' & \cdots & C_K + D_KSD_K' & -D_KS \\ -SD_1' & -SD_2' & \cdots & -SD_K' & S \end{bmatrix}$$

where

$S = (\sum G_k' R_k G_k)^{-1}$ k runs over data blocks $k=1, 2, \ldots K, K+1$;

$C_k = (X_k'X_k)^{-1}$ for $k=1, 2, \ldots K$,

$D_k = (X_k'X_k)^{-1}X_k' G_k$ for $k=1, 2, \ldots K$.

Image 4:

Fastest possible computation of the Minimum Norm Quadratic Unbiased Estimates (MINQUE) :

vector $\{\sigma^2_i\} = [\sigma^2_1, \sigma^2_2, \ldots, \sigma^2_n]^T = \mathbf{F}^{-1}\,\mathbf{q}$

where

n = number of observed carrier-phases

$\mathbf{q}$ = vector $\{q_i\}$ = vector $\{\mathbf{y}^T \mathbf{R}\, T_i\, \mathbf{y}\}$

$\mathbf{F}$ = matrix $\{f_{i,j}\}$ = matrix $\{\mathrm{tr}\, \mathbf{R}\, T_i\, \mathbf{R}\, T_j\}$

T_i = diagonal matrix $(\delta^1_i, \delta^2_i, \ldots, \delta^N_i)$

N = total number of all observations

In case of uncorrelated measurements of a scalar variable this MINQUE solution would collapse into the simple formula for computing the error variance of a mean: σ^2/n

$$\mathbf{R} = \mathbf{I} - \begin{bmatrix} X_1 & & & G_1 \\ & X_2 & & G_2 \\ & & \ddots & \vdots \\ & & X_K & G_K \end{bmatrix} \begin{bmatrix} C_1 + D_1SD_1^T & D_1SD_2^T & \cdots & D_1SD_K^T & -D_1S \\ D_2SD_1^T & C_2 + D_2SD_2^T & \cdots & D_2SD_K^T & -D_2S \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_KSD_1^T & D_KSD_2^T & & C_K + D_KSD_K^T & -D_KS \\ -SD_1^T & -SD_2^T & \cdots & -SD_K^T & S \end{bmatrix} \begin{bmatrix} X_1 & & & G_1 \\ & X_2 & & G_2 \\ & & \ddots & \vdots \\ & & X_K & G_K \end{bmatrix}^T$$

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