Optimal Kalman Filtering for ultra-reliable Tracking

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INTRODUCTION

The theory of optimal Kalman Filters (KF) provides means for atmospheric remote sensing that can make precision tracking of an air surveillance system to work as reliably as possible. The theory requires that Kalman's observability and controllability conditions are satisfied which, however, appears to be quite tricky to be continuously verified and maintained. The error covariance matrix of a state vector provides the most practical means for monitoring the stability of optimal Kalman Filtering if only it can be reliably estimated in real-time. However, this is much more demanding than estimating the state vector. Error propagation modeling is normally used for estimating accuracies. Unfortunately, this is unreliable because the two stability conditions above are not sufficient to guarantee the filtering stability when modeling errors exist.

Firstly, all incoming data must have a sufficient degree of overdetermination. Secondly, it only is an observed internal consistency of the measurements that can provide adequate information for any reliable estimation of accuracies. The computations of such estimation methods are being developed by making use of the geodetic Helmert-Wolf formulas [1] and C. R. Rao's theory [2] of Minimum Norm Quadratic Unbiased Estimation (MINQUE). The precisely computed residuals (i.e. innovation sequences) from the semi-analytic Helmert-Wolf inversion method [3] seem to offer a suitable real-time source of data for computing the most reliable estimates of all different systematic and random error components. These parameters are absolutely required by optimal Kalman Filtering. The confidence intervals can then be immediately computed for all state parameters and the filtering stability submitted under ultra-reliable control in real-time.

OPTIMAL KALMAN FILTERING

Equations (1) **-** (2) below describe the time behavior of an optimal Kalman Filter (KF) system to be used for the real-time fusion and processing all available data. Equation (1) tells how a measurement vector $\mathbf{y}_t$ depends on a state vector $\mathbf{s}_t$ and on a random error vector $\mathbf{e}_t$ at each time-point t as well as on a (more or less) constant calibration drift and systematic error vector $\mathbf{c}$. This is the linearized Measurement Equation:

$$\mathbf{y}_t = H_t\,\mathbf{s}_t + F^y{}_t\,\mathbf{c} + \mathbf{e}_t \quad \text{for } t = 1, 2, ... \tag{1}$$

where the matrices H_t and $F^y{}_t$ are the Jacobians that stem from the partial derivatives of dependencies between the measurements $\mathbf{y}_t$ and the unknown states $\mathbf{s}_t$ and the various calibration drifts and systematic errors $\mathbf{c}$ which all are to be sensed remotely. Time evolution of the overall system is described by the linearized System Equation:

$$\mathbf{s}_t = A_t\,\mathbf{s}_{t-1} + B_t\,\mathbf{u}_{t-1} + F^s{}_t\,\mathbf{c} + \mathbf{a}_t \quad \text{for } t = 1, 2, ... \tag{2}$$

where matrix A_t is the state transition matrix, B_t is the control gain matrix and $F^s{}_t$ tells how the system depends on the calibration drifts and systematic errors $\mathbf{c}$. Equation (2) tells how a present state vector $\mathbf{s}_t$ of the overall system develops from its previous states $\mathbf{s}_{t-1}$ when also affected by known controls $\mathbf{u}_{t-1}$ and random noises $\mathbf{a}_t$.

Optimal Kalman Filter estimates of the consequtive state vectors $\mathbf{s}_t$, $\mathbf{s}_{t-1}$, $\mathbf{s}_{t-2}$, … , $\mathbf{s}_{t-K-1}$ and all the unknown constant calibration drifts and systematic errors **c** are computed by using the Helmert-Wolf formulas:

$$\left.\begin{aligned} \mathbf{s}_{t-l} &= \{X_{t-l}'\, V_{t-l}^{-1}\, X_{t-l}\}^{-1}\, X_{t-l}'\, V_{t-l}^{-1}\, (\mathbf{y}_{t-l} - G_{t-l}\,\mathbf{c}) \\ \mathbf{c} &= \{\Sigma\, X_{t-l}'\, R_{t-l}\, X_{t-l}\}^{-1}\, \Sigma\, G_{t-l}' R_{t-l}\, \mathbf{y}_{t-l} \end{aligned}\right\} \quad (3)$$

where the summation index l (= 0, 1, 2, …, K-2, K-1) runs over a sufficiently long (=K) time-series of observational data so that all the calibration drifts and systematic errors **c** become observable. This observability has now the simple meaning that all error variances of the state parameters just stay within acceptable tolerances. For an explanation of the different symbols in (3) and (4), see [4] or [5] which papers also explain Lange's Precision Matrix (LPM) for a semi-analytic piecewise computing of the following huge matrix of the error variances and covariances:

$$\mathrm{Cov}(\mathbf{s}_t, \mathbf{s}_{t-1}, \mathbf{s}_{t-2}, \ldots, \mathbf{s}_{t-K-1}, \mathbf{c}) = \quad (4)$$

$$\begin{bmatrix} C_1 + D_1SD_1' & D_1SD_2' & \cdots & D_1SD_K' & -D_1S \\ D_2SD_1' & C_2 + D_2SD_2' & \cdots & D_2SD_K' & -D_2S \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_KSD_1' & D_KSD_2' & & C_K + D_KSD_K' & -D_KS \\ -SD_1' & -SD_2' & \cdots & -SD_K' & S \end{bmatrix}$$

The measurement errors $\mathbf{e}_t$, $\mathbf{e}_{t-1}$, $\mathbf{e}_{t-2}$, … , $\mathbf{e}_{t-K-1}$ and system noises $\mathbf{a}_t$, $\mathbf{a}_{t-1}$, $\mathbf{a}_{t-2}$, … , $\mathbf{a}_{t-K-1}$ must have no components that may auto- or cross-correlate in an unknown way less the optimality of Kalman Filtering will be lost. Thus, all the underlying variance components are estimated by the MINQUE method [2] and taken into account by factoring them with the help of the matrices F^y_t and F^s_t in (1) and (2). These two matrices should primarily be obtained from known physicals dependencies between the various error components. As completely unknown dependencies may also exist their models can be identified by using the Singular Value Decomposition (SVD) and the (generalised) Canonical Correlation techniques, see [6] and [7], respectively.

RELIABLE ACCURACY ESTIMATION

The error variances of different sensor readings can be reliably estimated only from their observed internal consistency by using e.g. the Minimum Norm Quadratic Unbiased Estimation (MINQUE) theory [2]. Fortunately, the GNSS signals and all such satellite systems with their augmentations render a surplus of overdetermination which makes it possible to estimate all added parameters that stem from both various calibration drifts and random errors. Significant synergy advantages often come from integrating different subsystems. However, long moving windows of data may also be required on a continuous basis or extremely long windows on a temporal basis (i.e. for so-called training of the filter). The very demanding computations of [8] are in this way extended also to optimal Adaptive Kalman Filtering (AKF) under the general title of Fast Kalman Filtering (FKF), see [5].

Fig. 1. F.R. Helmert

Fig. 2. H. Wolf

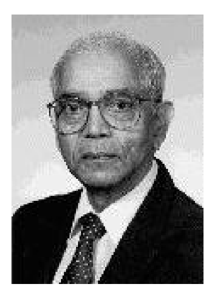

Fig. 3. C.R. Rao

CONCLUDING REMARKS

An optimal data filtering system is needed to provide reliable information on all the measured parameters like signal propagation time, signal angle of arrival, Doppler shift and signal amplitude etc. Optimal Kalman Filtering provides a stable method for repeatedly updating a vector of weighted mean values (probe coordinates, calibration drifts and systematic model errors) by an optimal exploitation of all input data in real-time. This is not necessarily true with other control methods such as fuzzy logic.

The semi-analytical computing method of FKF provides the possibility of processing huge amounts of input data in real-time with ultra-reliable accuracy estimation [9]. The obvious benefits of the Statistical Calibration method [5] come from practical life, as there usually are large amounts of sensors (radars, transponders, profilers, GPS- receivers, etc.) that operate far from each other, without possibilities for immediate maintenance and/or precise physical recalibration.

These fast computational solutions in [1], [2], [4] and [7] are being implemented in the GAMIT/GLOBK GPS software package of the Massachusetts Institute of Technology (MIT).

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